# ALABAMA MATH PARTICIPANT'S ENGAGEMENT GUIDE

Doing Math at Home: A Family Engagement Session

# Kindergarten Mathematics Overview

Kindergarten content is organized into six Alabama Content Areas as outlined in the table below: Foundations of Counting; Operations and Algebraic Thinking; Operations with Numbers: Base Ten; Data Analysis; Measurement; and Geometry.  Related standards are grouped into clusters, which are listed below each content area. Standards indicate what the student should know or be able to do by the end of the grade.

| Alabama Content Areas | Foundations of Counting | Operations and Algebraic Thinking | Operations with Numbers: Base Ten | Data Analysis | Measurement | Geometry |
|---|---|---|---|---|---|---|
| Clusters | • Know number names and the count sequence.<br>• Count to tell the number of objects.<br>• Compare numbers. | • Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.<br>• Understand simple patterns. | • Work with numbers 11-19 to gain foundations for place value. | • Collect and analyze data and interpret results. | • Describe and compare measurable attributes. | • Identify and describe shapes.<br>• Analyze, compare, create, and compose shapes. |

The eight Student Mathematical Practices, listed in the chart below, represent what students are doing as they learn mathematics.  Students should regularly engage in these processes and proficiencies at every level throughout their mathematical studies.  Proficiency with these practices is critical in using mathematics, in the classroom and in everyday life.  **The Student Mathematical Practices are standards to be incorporated across all grades**.

| Student Mathematical Practices | |
|---|---|
| 1. Make sense of problems and persevere in solving them. | 5. Use appropriate tools strategically. |
| 2. Reason abstractly and quantitatively. | 6. Attend to precision. |
| 3. Construct viable arguments and critique the reasoning of others. | 7. Look for and make use of structure. |
| 4. Model with mathematics. | 8. Look for and express regularity in repeated reasoning. |

# Content Priorities

In kindergarten, instructional time should focus on two critical areas:
1.  developing a sound sense of numbers by representing and comparing numbers, using sets of objects; and
2.  recognizing and describing shapes and using spatial relations.

# Grade 1 Mathematics Overview

Grade 1 content is organized into five Alabama Content Areas as outlined in the table below: Operations and Algebraic Thinking; Operations with Numbers: Base Ten; Data Analysis; Measurement; and Geometry. Related standards are grouped into clusters, which are listed below each content area. Standards indicate what the student should know or be able to do by the end of the grade.

| Alabama Content Areas | Operations and Algebraic Thinking | Operations with Numbers: Base Ten | Data Analysis | Measurement | Geometry |
|---|---|---|---|---|---|
| **Clusters** | • Represent and solve problems involving addition and subtraction.<br>• Understand and apply properties of operations and the relationship between addition and subtraction.<br>• Add and subtract within 20.<br>• Work with addition and subtraction equations.<br>• Understand simple patterns. | • Extend the counting sequence.<br>• Understand place value.<br>• Use place value understanding and properties of operations to add and subtract. | • Collect and analyze data and interpret results. | • Describe and compare measurable attributes.<br>• Work with time and money. | • Reason with shapes and their attributes. |

The eight Student Mathematical Practices listed in the chart below represent what students are doing as they learn mathematics. Students should regularly engage in these processes and proficiencies at every level throughout their mathematical studies. Proficiency with these practices is critical in using mathematics, both in the classroom and in everyday life. **The Student Mathematical Practices are standards to be incorporated across all grades**.

| Student Mathematical Practices | |
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| 1. Make sense of problems and persevere in solving them. | 5. Use appropriate tools strategically. |
| 2. Reason abstractly and quantitatively. | 6. Attend to precision. |
| 3. Construct viable arguments and critique the reasoning of others. | 7. Look for and make use of structure. |
| 4. Model with mathematics. | 8. Look for and express regularity in repeated reasoning. |

# Content Priorities

In Grade 1, instructional time should focus on four critical areas:
1. developing understanding of addition, subtraction, and strategies for addition and subtraction within 20;
2. developing understanding of whole number relationships and place value, including grouping in tens and ones;
3. developing understanding of linear measurement and measuring lengths as iterating length units; and
4. reasoning about attributes of and composing and decomposing geometric shapes.

# Grade 2 Mathematics Overview

Grade 2 content is organized into five Alabama Content Areas as outlined in the table below: Operations and Algebraic Thinking; Operations with Numbers: Base Ten; Data Analysis; Measurement; and Geometry. Related standards are grouped into clusters, which are listed below each content area. Standards indicate what the student should know or be able to do by the end of the grade.

| Alabama Content Areas | Operations and Algebraic Thinking | Operations with Numbers: Base Ten | Data Analysis | Measurement | Geometry |
|---|---|---|---|---|---|
| Clusters | • Represent and solve problems involving addition and subtraction.<br>• Add and subtract within 20.<br>• Work with equal groups of objects to gain foundations for multiplication.<br>• Understand simple patterns. | • Understand place value.<br>• Use place value understanding and properties of operations to add and subtract. | • Collect and analyze data and interpret results. | • Measure and estimate lengths in standard units.<br>• Relate addition and subtraction to length.<br>• Work with time and money. | • Reason with shapes and their attributes. |

The eight Student Mathematical Practices listed in the chart below represent what students are doing as they learn mathematics. Students should regularly engage in these processes and proficiencies at every level throughout their mathematical studies. Proficiency with these practices is critical in using mathematics in the classroom and in everyday life. **The Student Mathematical Practices are standards which should be incorporated across all grades**.

| Student Mathematical Practices | |
|---|---|
| 1. Make sense of problems and persevere in solving them. | 5. Use appropriate tools strategically. |
| 2. Reason abstractly and quantitatively. | 6. Attend to precision. |
| 3. Construct viable arguments and critique the reasoning of others. | 7. Look for and make use of structure. |
| 4. Model with mathematics. | 8. Look for and express regularity in repeated reasoning. |

# Content Priorities

In Grade 2, instructional time should focus on four critical areas:
1. building fluency with addition and subtraction;
2. extending understanding of base-ten notation;
3. using standard units of measure; and
4. describing and analyzing shapes.

# Grade 3 Mathematics Overview

Grade 3 content is organized into six Alabama Content Areas of study as outlined in the table below: Operations and Algebraic Thinking; Operations with Numbers: Base Ten; Operations with Numbers: Fractions; Data Analysis; Measurement; and Geometry. Related standards are grouped into clusters, which are listed below each content area. Standards indicate what the student should know or be able to do by the end of the grade.

| Alabama Content Areas | Operations and Algebraic Thinking | Operations with Numbers: Base Ten | Operations with Numbers: Fractions | Data Analysis | Measurement | Geometry |
|---|---|---|---|---|---|---|
| **Clusters** | • Represent and solve problems involving multiplication and division.<br>• Understand properties of multiplication and the relationship between multiplication and division.<br>• Multiply and divide within 100.<br>• Solve problems involving the four operations, and identify and explain patterns in arithmetic. | • Use place value understanding and properties of operations to perform multi-digit arithmetic. | • Develop understanding of fractions as numbers. | • Represent and interpret data. | • Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.<br>• Geometric measurement: understand concepts of area and relate area to multiplication and addition.<br>• Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures. | • Reason with shapes and their attributes |

The eight Student Mathematical Practices listed in the chart below represent what students are doing as they learn mathematics. Students should regularly engage in these processes and proficiencies at every level throughout their mathematical studies. Proficiency with these practices is critical in using mathematics, both in the classroom and in everyday life. **The Student Mathematical Practices should be regarded as standards to be incorporated across all grades.**

| Student Mathematical Practices | |
|---|---|
| 1. Make sense of problems and persevere in solving them. | 5. Use appropriate tools strategically. |
| 2. Reason abstractly and quantitatively. | 6. Attend to precision. |
| 3. Construct viable arguments and critique the reasoning of others. | 7. Look for and make use of structure. |
| 4. Model with mathematics. | 8. Look for and express regularity in repeated reasoning. |

# Content Priorities

In Grade 3, instructional time should focus on four critical areas:
1. developing understanding of multiplication and division and strategies for multiplication and division within 100;
2. developing understanding of fractions, especially unit fractions (fractions with numerator 1);
3. developing understanding of the structure of rectangular arrays and of area; and
4. describing and analyzing two-dimensional shapes.

# Grade 4 Mathematics Overview

Grade 4 content is organized into six Alabama Content Areas outlined in the table below: Operations and Algebraic Thinking; Operations with Numbers: Base Ten; Operations with Numbers: Fractions; Data Analysis; Measurement; and Geometry. Related standards are grouped into clusters, which are listed below each content area. Standards indicate what the student should know or be able to do by the end of the grade.

| Alabama Content Areas | Operations and Algebraic Thinking | Operations with Numbers: Base Ten | Operations with Numbers: Fractions | Data Analysis | Measurement | Geometry |
|---|---|---|---|---|---|---|
| **Clusters** | • Gain familiarity with factors and multiples.<br>• Solve problems with whole numbers using the four operations.<br>• Generate and analyze patterns. | • Generalize place value understanding for multi-digit whole numbers.<br>• Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers. | • Extend understanding of fraction equivalence and ordering.<br>• Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.<br>• Understand decimal notation for fractions, and compare decimal fractions. | • Represent and interpret data. | • Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.<br>• Geometric measurement: understand concepts of angles and measure angles. | • Draw and identify lines and angles, and identify shapes by properties of their lines and angles. |

The eight Student Mathematical Practices listed in the chart below represent what students are doing as they learn mathematics. Students should regularly engage in these processes and proficiencies at every level throughout their mathematical studies. Proficiency with these practices is critical in using mathematics, both within the classroom and in life. **The Student Mathematical Practices are as standards which should be incorporated across all grades.**

| Student Mathematical Practices | |
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| 1. Make sense of problems and persevere in solving them. | 5. Use appropriate tools strategically. |
| 2. Reason abstractly and quantitatively. | 6. Attend to precision. |
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| 4. Model with mathematics. | 8. Look for and express regularity in repeated reasoning. |

# Content Priorities

In Grade 4, instructional time should focus on three areas:
1. developing understanding and fluency with multi-digit multiplication, and understanding of division to find quotients involving multi-digit dividends;
2. developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; and
3. understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, or symmetry.

**2019 Alabama Course of Study: Mathematics**

# Grade 5 Mathematics Overview

Grade 5 content is organized into six Alabama Content Areas as outlined in the table below: Operations and Algebraic Thinking; Operations with Numbers: Base Ten; Operations with Numbers: Fractions; Data Analysis; Measurement; and Geometry. Related standards are grouped into clusters, which are listed below each content area. Standards indicate what the student should know or be able to do by the end of the grade.

| Alabama Content Areas | Operations and Algebraic Thinking | Operations with Numbers: Base Ten | Operations with Numbers: Fractions | Data Analysis | Measurement | Geometry |
|---|---|---|---|---|---|---|
| **Clusters** | • Write and interpret numerical expressions.<br>• Analyze patterns and relationships. | • Understand the place value system.<br>• Perform operations with multi-digit whole numbers and decimals to hundredths. | • Use equivalent fractions as a strategy to add and subtract fractions.<br>• Apply and extend previous understandings of multiplication and division to multiply and divide fractions. | • Represent and interpret data. | • Convert like measurement units within a given measurement system.<br>• Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition. | • Graph points on the coordinate plane to solve real-world and mathematical problems.<br>• Classify two-dimensional figures into categories based on their properties. |

The eight Student Mathematical Practices listed in the chart below represent what students are doing as they learn mathematics. Students should regularly engage in these processes and proficiencies at every level throughout their mathematical studies. Proficiency with these practices is critical in using mathematics, both in the classroom and in everyday life. **The Student Mathematical Practices are standards which should be incorporated across all grades.**

| Student Mathematical Practices | |
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| 1. Make sense of problems and persevere in solving them. | 5. Use appropriate tools strategically. |
| 2. Reason abstractly and quantitatively. | 6. Attend to precision. |
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| 4. Model with mathematics. | 8. Look for and express regularity in repeated reasoning. |

# Content Priorities

In Grade 5, instructional time should focus on three critical areas:
1. developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions);
2. extending division to 2-digit divisors, integrating decimals into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and
3. developing understanding of volume.

# Grade 6 Mathematics Overview

Grade 6 content is organized into five Alabama Content Areas as outlined below: Proportional Reasoning; Number Systems and Operations; Algebra and Functions; Data Analysis, Statistics, and Probability; and Geometry and Measurement. Related standards are grouped into clusters, which are listed below each content area. Resources to support the Grades 6-8 standards are in Appendix D. Standards indicate what students should know and be able to do by the end of the course.

| Alabama Content Areas | Proportional Reasoning | Number Systems and Operations | Algebra and Functions | Data Analysis, Statistics, and Probability | Geometry and Measurement |
|---|---|---|---|---|---|
| **Clusters** | • Develop an understanding of ratio concepts and use reasoning about ratios to solve problems. | • Use prior knowledge of multiplication and division to divide fractions. <br> • Compute multi-digit numbers fluently and determine common factors and multiples. <br> • Apply knowledge of the number system to represent and use rational numbers in a variety of forms. | • Apply knowledge of arithmetic to read, write, and evaluate algebraic expressions. <br> • Use equations and inequalities to represent and solve real-world or mathematical problems. <br> • Identify and analyze relationships between independent and dependent variables. | • Use real-world and mathematical problems to analyze data and demonstrate an understanding of statistical variability and measures of center. | • Graph polygons in the coordinate plane to solve real-world and mathematical problems. <br> • Solve real-world and mathematical problems to determine area, surface area, and volume. |

The eight Student Mathematical Practices listed in the chart below represent what students are doing as they learn mathematics. Students should regularly engage in these processes and proficiencies at every level throughout their mathematical studies. Proficiency with these practices is critical in using mathematics, both in the classroom and in everyday life. **The Student Mathematical Practices are standards to be incorporated across all grades**.

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# Content Priorities

In Grade 6, instructional time should focus on five essential areas:
1. connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems;
2. completing understanding of division of fractions and extending the understanding of number sense to the system of rational numbers, including signed numbers;
3. writing, interpreting, and using expressions and equations; and
4. developing understanding of statistical thinking.

# Grade 7 Mathematics Overview

In Grade 7, content is organized into five Alabama Content Areas as outlined below: Proportional Reasoning; Number Systems and Operations; Algebra and Functions; Data Analysis, Statistics and Probability; and Geometry and Measurement. Related standards are grouped into clusters, which are listed below each content area. Resources to support the Grade 7 mathematical standards are in Appendix D. Standards indicate what students should know or be able to do by the end of the course.

| Alabama Content Areas | Proportional Reasoning | Number Systems and Operations | Algebra and Functions | Data Analysis, Statistics, and Probability | Geometry and Measurement |
|---|---|---|---|---|---|
| **Clusters** | • Analyze proportional relationships and use them to solve real-world and mathematical problems. | • Apply and extend prior knowledge of addition, subtraction, multiplication, and division to operations with rational numbers. | • Create equivalent expressions using the properties of operations.<br>• Solve real-world and mathematical problems using numerical and algebraic expressions, equations, and inequalities. | • Make inferences about a population using random sampling.<br>• Make inferences from an informal comparison of two populations.<br>• Investigate probability models. | • Construct and describe geometric figures, analyzing relationships among them.<br>• Solve real-world and mathematical problems involving angle measure, circumference, area, surface area, and volume. |

The eight Student Mathematical Practices listed in the chart below represent what students are doing as they learn mathematics. Students should regularly engage in these processes and proficiencies at every level throughout their mathematical studies. Proficiency with these practices is critical in using mathematics, both in the classroom and in everyday life. **The Student Mathematical Practices are standards to be incorporated across all grades**.

| Student Mathematical Practices | |
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| 1. Make sense of problems and persevere in solving them. | 5. Use appropriate tools strategically. |
| 2. Reason abstractly and quantitatively. | 6. Attend to precision. |
| 3. Construct viable arguments and critique the reasoning of others. | 7. Look for and make use of structure. |
| 4. Model with mathematics. | 8. Look for and express regularity in repeated reasoning. |

# Content Priorities

In Grade 7, instructional time should focus on four essential areas, all of which have equal importance:

1. developing understanding of and applying proportional relationships;
2. developing understanding of operations with rational numbers and working with expressions and linear equations;
3. solving problems involving scale drawings and informal geometric construction, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and
4. drawing inferences about populations based on samples.

# Grade 8 Mathematics Overview

In Grade 8, content is organized into four Alabama Content Areas outlined below: Number Systems and Operations; Algebra and Functions; Data Analysis, Statistics, and Probability; and Geometry and Measurement. Related standards are grouped into clusters, which are listed below each content area. Resources to support the Grade 8 mathematical standards are in Appendix D. Standards indicate what students should know or be able to do by the end of the course.

| Alabama Content Areas | Number Systems and Operations | Algebra and Functions | Data Analysis, Statistics, and Probability | Geometry and Measurement |
|---|---|---|---|---|
| **Clusters** | • Understand that the real number system is composed of rational and irrational numbers. | • Apply concepts of rational and integer exponents. <br> • Analyze the relationship between proportional and non-proportional situations. <br> • Analyze and solve linear equations and systems of two linear equations. <br> • Explain, evaluate, and compare functions. <br> • Use functions to model relationships between quantities. | • Investigate patterns of association in bivariate data. | • Understand congruence and similarity using physical models or technology. <br> • Analyze parallel lines cut by a transversal. <br> • Understand and apply the Pythagorean Theorem. <br> • Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. |

The eight Student Mathematical Practices listed in the chart below represent what students are doing as they learn mathematics. Students should regularly engage in these processes and proficiencies at every level throughout their mathematical studies. Proficiency with these practices is critical in using mathematics, both in the classroom and in everyday life. **The Student Mathematical Practices are standards to be incorporated across all grades**.

| Student Mathematical Practices | |
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| 1. Make sense of problems and persevere in solving them. | 5. Use appropriate tools strategically. |
| 2. Reason abstractly and quantitatively. | 6. Attend to precision. |
| 3. Construct viable arguments and critique the reasoning of others. | 7. Look for and make use of structure. |
| 4. Model with mathematics. | 8. Look for and express regularity in repeated reasoning. |

# Content Priorities

In Grade 8, instructional time should focus on three critical areas, all of which have equal importance:

1. construct and reason about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations;
2. describe the concept of a function and use functions to interpret quantitative relationships; and
3. analyze two- and three-dimensional figures and understand and apply the Pythagorean Theorem.

# Math Milestones — Kindergarten

**K:1**
How many blocks?
[Student tells how many.]

[Teacher slowly rearranges.]
If you count the blocks, how many do you think there will be?

**K:2**
There are 4 books on the floor
and 6 books on the bed.
How many books are there?

**K:3**
Say the counting numbers. Also say the missing numbers.

☞ _9_  _10_  _11_  ____  ____  _14_

☞ _55_  _56_  _57_  _58_  _59_  ____

**K:4**
Are both of the bears correct?
[Student uses manipulatives to answer.]

"There are 3 squares."

"These two triangles can be put together to make a new triangle."

**K:5**
[Teacher puts 3 red counters on table.]
Put some blue counters here to make 10 counters in all. [Student completes this task.] How many counters did you add?
[Student determines the answer.]

Write the missing number: 3 + ____ = 10

**K:6**
Are there more shells or more sea stars?

**K:7**
Hazel told a story. Write or say two numbers that will make Hazel's story true.

I have 10 pennies in my hands.

I have _____ pennies in my left hand.

I have _____ pennies in my right hand.

What other numbers will also make Hazel's story true?

**K:8**
[Teacher holds out 5 paper clips.]
How many do I have?
[Student counts the paper clips.]
[Teacher puts both hands behind back, then brings out 0, 1, 2, 3, 4, or 5 paper clips in one hand.] How many are in this hand?
[Student counts the paper clips.]
How many are in my other hand?

**K:9**
# 6   5
Point to the greater number. [Student points.] Tell me how you decided.

**K:10**
5 dogs were playing.
Then 3 more dogs came.
How many dogs are here now?

**K:11**
9 birds were in a tree.
5 birds flew away.
How many birds are there now?

**K:12**
Draw 16 circles. Use a [favorite color] marker for 10 of them. Use a pencil for the rest. [Student draws.]
How many are [favorite color]? How many are in pencil?
Write the missing number: 16 = 10 + ____

**K:13**
Write or say the missing numbers.
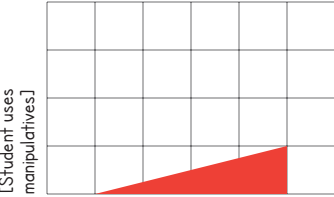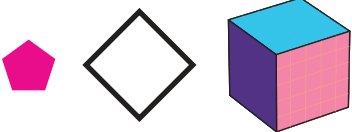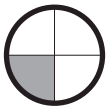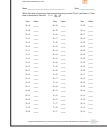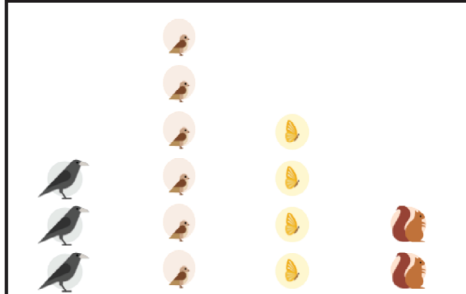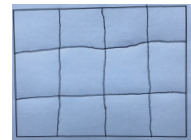
3 + 1 = _____        2 + 3 = _____
5 + 0 = _____        2 − 2 = _____
4 − 3 = _____        5 − 3 = _____

**K:14**
Are there more land animals or more sea animals?

elephant    clownfish    gorilla
dolphin     mantis       snake
seahorse    octopus      shark

# Math Milestones — Grade 1

**1:1** 10 lions were at the water hole. 4 lions joined them. Then 3 more lions joined. How many lions were at the water hole after that?

**1:2** True or false?

6 tens + 4 ones < 4 ones + 7 tens

7 ones + 5 tens = _____

**1:3** Using a paper clip as a unit of length, draw a straight line 7 units long.

**1:4** Our class watched the weather for 21 days. On a chart, we marked each day as one of three kinds: sunny, cloudy, or rainy.

| Sunny | Cloudy | Rainy |
|---|---|---|
| 卌 | 卌 卌 II | 卌 IIII |

(1) Count all the tally marks. Does your answer make sense?

(2) How many days were not rainy?

(3) Now create your own question by circling one word. Use the data to answer your question.

How many more cloudy/rainy days were
(circle one word)
there than sunny days?

**1:5** Tyler has 6 more grapes than Zoey. Zoey has 8 grapes. How many grapes does Tyler have?

Equation model: _____

Answer: Tyler has _____ grapes.

**1:6** I have 24 straws in a jar. I have 30 straws in a bag. How many straws do I have?

**1:7** If the class works hard, our teacher will put a marble in a jar. We will have a party when there are 10 marbles in the jar. Today there are 6 marbles in the jar. How many marbles do we need for a party?

**1:8**

90 − 40 = _____

9 apples − 4 apples = _____ _____
(number)    (unit)

9 cups − 4 cups = _____ _____
(number)    (unit)

9 tens − 4 tens = _____ _____
(number)    (unit)

**1:9** Write the missing numbers.

4 + 5 = _____     7 − 4 = _____

10 − 8 = _____     2 + 6 = _____

4 + _____ = 10     7 + _____ = 10

**1:10**

Write the sum.
$$\begin{array}{r} 37 \\ + 46 \\ \hline \end{array}$$

**1:11** Write the missing numbers. Tell how you got the answers.

8 + 5 = _____          8 − _____ = 2

13 − 4 = _____          _____ − 5 = 4

7 + 4 = 10 + _____          6 + _____ = 12

**1:12** Grace tried to blow out 15 candles on her birthday cake. Grace blew out 9 candles. How many candles are still lit?

Equation model: _____

Answer: _____ candles are still lit.

**1:13** When I fell asleep last night, there were 8 icicles outside my window. When I woke up this morning, there were 3 icicles. How many icicles fell while I slept?

**1:14** One statement below is false. Find the false statement. How did you decide?

[Student uses manipulatives]

A square can be created using triangles like this one.

None of these are squares.

The shaded part of the circle is one fourth of the whole circle.

# Math Milestones — Grade 2

**2:1** Avi made a paper chain. Then Avi added 29 more links to the paper chain. Now there are 52 links in the paper chain. How many links were in the paper chain before?

**2:2** (1) True or false?

(a) 2 hundreds + 3 ones > 5 tens + 9 ones

(b) 9 tens + 2 hundreds + 4 ones < 924

(c) 456 < 5 hundreds

(2) Write the number that makes each statement true.

(a) 7 ones + 5 hundreds = _____

(b) 14 tens = _____

(c) 90 + 300 + 4 = _____

**2:3** Write the sums and differences.

$$36 + 45 \qquad 72 - 17 \qquad 64 + 27 \qquad 82 - 55$$

**2:4** Faith went to the park. The picture graph shows all of the animals Faith saw.

1 crow    1 sparrow    1 butterfly    1 squirrel

Faith said, "I saw fewer butterflies than birds." How many fewer butterflies did Faith see?

**2:5** Write the value of each sum. Use as much time as you need. If you "just knew it," then draw a check mark, like this: 2 + 2  $4\checkmark$

Click here for student handout 2:5

**2:6** A rope is 32 feet long. The rope is cut into two pieces. One piece is 3 feet long. How long is the other piece?

Equation model: _____

Answer: _____ feet

**2:7** (1) Write the number that makes the statement true.

6 hundreds + 3 tens + 4 ones

= 5 hundreds + _____ tens + 4 ones.

(2) How do you know your statement is true?

(3) Look for connections between your statement and this subtraction problem. What connections can you see?

$$\overset{5\ 13}{\cancel{634}} - 482 = 152$$

**2:8** Write the number that makes each equation true. Use as much time as you need.

Click here for student handout 2:8

**2:9** A farmer said, "Last night some deer came and ate 16 of my cabbages. Now I only have 38 cabbages." How many cabbages were there before the deer came?

Equation model: _____

Answer: There were _____ cabbages.

**2:10** Check the subtraction by adding.

946 − 678 = 268

**2:11** A grass snake is 28 inches long. A rat snake is 74 inches long. How much longer is the rat snake?

Draw a diagram to illustrate your solution. Label the diagram with numbers.

**2:12** At recess there was a jump-rope contest.

Leslie: I won because I jumped 25 more times than Catherine.

I jumped 81 times.

How many times did Catherine jump?

Equation model: _____

Answer: Catherine jumped _____ times.

**2:13** Marlon and Malia went apple-picking.

Marlon: I picked 12 apples.

Malia: You picked 13 fewer apples than I did.

How many apples did Malia pick?

Equation model: _____

Answer: Malia picked _____ apples.

**2:14** Zariah got one answer wrong.

(1) Which answer did Zariah get wrong?

(2) Correct Zariah's wrong answer.

(a) Show how the rectangle can be divided into 15 squares.

(b) **2** _____ halves make one whole.

(c) Draw a triangle. All three sides of your triangle must have different lengths.
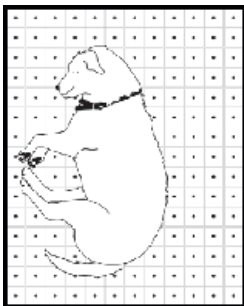
# Math Milestones — Grade 3

**3:1** There are five teams in the volleyball league. Every team has six players. How many players are in the volleyball league?

Equation model: _____

Answer: _____

**3:2**



The picture shows a dog sleeping on a rug. The rug design is a rectangular array of squares with a dot in each square.

Below are four expressions. One expression equals the total number of dots in the rug design. Which expression equals the total number of dots in the rug design? Tell how you decided.

$12 \times 14, \ 11 \times 14, \ 12 \times 15, \ 11 \times 15$

**3:3** **(1)** How much area is shaded?



— 1 unit of length

**(2)** Using a ruler, draw a rectangle with area 28 square centimeters. Write the length and width of your rectangle.

Length: _____    Width: _____

**3:4** Jasmine bought 45 corn seeds. She arranged the seeds into piles of 9 seeds each. How many piles were there?

Equation model: _____

Answer: _____

**3:5** Our class picked up litter on the playground. One student wrote tally marks to record the things we picked up.

Paper    卌 卌 卌 卌
Plastic  卌 卌 卌 卌 卌 卌  卌 卌
Glass    卌 卌
Garbage  卌 卌 卌 卌 卌 卌

Show the data another way by drawing a scaled picture graph in which 1 picture stands for 10 things picked up.

**3:6** Using what you know about fractions, decide which is greater, $\frac{1}{73}$ or $\frac{1}{41}$. Tell how you decided.

**3:7** Here is a list of numbers. Where does each number belong on the number line?

$$\frac{1}{2}, \ \frac{1}{4}, \ 2, \ \frac{5}{4}, \ \frac{2}{2}, \ \frac{3}{2}, \ \frac{6}{4}, \ \frac{2}{4}, \ \frac{3}{1}$$

Draw a dot to show the location of each number. Label each dot. The first number in the list has been located for you.



**3:8** **(1)** Name two attributes that are shared by triangles and squares.

**(2)** Name a category of shapes that includes triangles and squares and also includes other shapes that have both of the attributes you named.

**3:9** Our class painted pictures. The teacher will hang the pictures on 4 bulletin boards. The teacher will hang the same number of pictures on each board. How many pictures will be on each board? There are 32 pictures to hang.

**3:10** Alice forgot what $7 \times 8$ equals. Alice knows that $5 \times 8 = 40$ and $2 \times 8 = 16$.

**(1)** Write a sentence to tell Alice how she can find the value of $7 \times 8$ by using the two facts she knows.

**(2)** Draw a diagram that could help Alice understand why your method works.

**(3)** Choose two numbers other than 7 and 8, and try using your method to multiply them. Will your method work for any pair of factors? Say why you think so.

**3:11** Steven, Hawa, and 4 more friends went to the park. Steven brought 24 water balloons. Hawa brought 24 water balloons. Nobody else brought water balloons. The 6 friends shared all the water balloons equally. How many water balloons did each friend get?

**3:12** Write the value of each product. Use as much time as you need. If you "just knew it," then draw a check mark, like this:

$2 \times 2$ ___4 ✓___

Click here for student handout 3:12

**3:13** Write the number that makes each equation true. Use as much time as you need.

Click here for student handout 3:13

**3:14** Write the sums and differences.

| With pencil and paper | | | | Mentally |
|---|---|---|---|---|
| 351 | 264 | 625 | 831 | $800 - 300$ |
| $+ 472$ | $+ 438$ | $- 261$ | $- 444$ | $240 + 540$ |
| | | | | $365 - 165$ |
| | | | | $612 - 13$ |

# Math Milestones — Grade 4

**4:1** A tablespoon holds 15 ml of olive oil, which is 3 times as much as a teaspoon holds. How many ml of olive oil does a teaspoon hold?

Equation model: _____

Answer: _____

**4:2**

? units     ? units    ? units

7 units | 700 square units | 210 sq units

49 square units
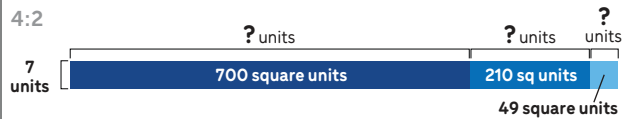
**(1)** Find the three missing lengths and write them on the diagram. Compare answers with a classmate.

**(2)** What is the total area of the diagram?

**(3)** Look for connections between the diagram and the division problem. What connections can you see?

$$7\overline{)959}$$

$$\begin{array}{r} 137 \\ 7\overline{)959} \\ -700 \\ \hline 259 \\ -210 \\ \hline 49 \\ -49 \\ \hline 0 \end{array}$$

**4:3** Everyone in class measured the length of their pencil. Here are the measurements:

(number line from 4 to 8 inches with x marks)
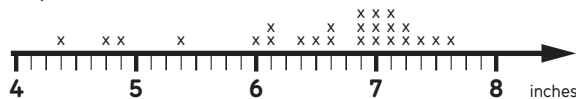
**(1)** How many pencils were measured?

**(2)** How much longer was the longest pencil than the shortest pencil?

**(3)** Could two of the pencils be laid end to end to make a total length of 1 foot?

**4:4**

**(1)** Compare $\frac{5}{9}$ to $\frac{4}{7}$. First do it by making equal denominators. Then do it by making equal numerators.

**(2)** Ariana said, "$\frac{300}{400}$ looks greater than $\frac{3}{4}$. How can they be the same size?" Write or say an explanation that could help Ariana understand why $\frac{300}{400}$ and $\frac{3}{4}$ are the same size.

**(3)** Which is closer to 1 on a number line, $\frac{4}{5}$ or $\frac{5}{4}$? Tell how you decided. Draw a number line and show $\frac{4}{5}$ and $\frac{5}{4}$ accurately on the number line.

**4:5**

**(1a–f)** Write the values of the products. Compare answers with a classmate.

(a) $4 \times \frac{1}{7} = $ _____

(b) $6 \times \frac{4}{7} = $ _____

(c) $86 \times \frac{1}{86} = $ _____

(d) $6 \times \frac{8}{2} = $ _____

(e) $9 \times \frac{1}{9} = $ _____

(f) $9 \times \frac{2}{9} = $ _____

**(1g)** Which answer is twice as much as the answer for (e)?

**(1h)** Which answer is six times as much as the answer for (a)?

**(1i)** Which two answers are equal?

**(2)** Zoe was reading her math book. She saw the equation $6 \times (4 + \frac{1}{2}) = 24 + 3$. She said, "I don't get it—where did the 24 and the 3 come from?" Write an explanation that could answer Zoe's question.

**4:6** Grandpa took a jar of pennies to the bank. He said, "I'd like nickels for this, please." The bank teller poured the pennies into a counting machine. "Eighty-seven dollars and forty-two cents," said the teller. **(1)** How many nickels did Grandpa get? **(2)** Check your answer with an estimate.

**4:7** Write the values of the expressions. Read each completed equation aloud.

3 fifths + 2 fifths = _____

$\frac{1}{10} + \frac{3}{100} = $ _____ (fraction)

= _____ (decimal)

$\frac{6}{25} + \frac{6}{25} = $ _____

$\frac{1}{8} + \frac{5}{8} - \frac{3}{8} = $ _____

**4:8** $L$ is a line, $R$ is a ray, and $T$ is a triangle. True or false:

**(1)** Line $L$ is a line of symmetry for triangle $T$.

**(2)** Line $L$ intersects ray $R$.

**(3)** Triangle $T$ has two angles measuring less than 90 degrees.
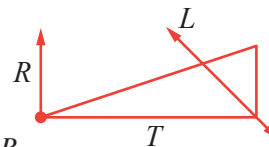
**4:9** In gym it was fitness day. Students ran laps around the gym.

Leslie: I ran $1\frac{2}{3}$ more laps than Catherine.

I ran $6\frac{1}{3}$ laps.

How many laps did Catherine run?

**4:10** Write the values of the products and quotients. Check the quotients by multiplying.

Mentally

$40 \times 20$
$30 \times 11$
$12 \times 60$
$5 \times 19$
$480 \div 8$

With pencil and paper

$\begin{array}{r} 6{,}132 \\ \times \quad 6 \\ \hline \end{array}$

$\begin{array}{r} 48 \\ \times 39 \\ \hline \end{array}$

$7\overline{)8{,}722}$

**4:11** A cook in the school kitchen uses 6 oz of cheese to make a pizza. The kitchen has 45 lb of cheese. How many pizzas will that make?

**4:12** The pickup truck can carry $1\frac{3}{5}$ tons. The super hauler truck can carry 300 times as much. How many tons can the super hauler truck carry?

Super Hauler Truck

Pickup Truck

**4:13**

**(1)** A red rectangle has length $L = 12$ in and width $W = 6$ in. Use the formula $A = L \times W$ to find the area of the red rectangle.

**(2)** A blue rectangle has length 1 ft and width $\frac{1}{2}$ ft. Draw a picture to show that two copies of the blue rectangle make one square foot. Based on your picture, what is the area of the blue rectangle?

**(3)** Do the red rectangle and the blue rectangle have equal areas? Tell how you decided.

**4:14** $540{,}909 + 87{,}808 - 5{,}864 + 2{,}556 = \,?$

# Math Milestones — Grade 5

**Math Milestones**

**5:1** A school needed 240 four-packs of juice boxes for a field trip. However, the school accidentally bought 240 *six-packs* of juice boxes. How many extra juice boxes did the school buy?

**5:2** After a hurricane, the 12 residents of a nursing home didn't have any clean water to drink. Their neighbors donated 40 gallons of bottled water, which would provide ____ gallons for each resident.

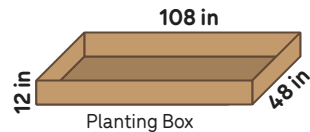**5:3** A neighborhood garden will have 6 wooden planting boxes. Every box will have the same shape (see diagram). Soil can be bought by the truckload; a truckload is 54 ft³ of soil. How many truckloads of soil will fill all of the boxes?

108 in
12 in
48 in
Planting Box

**5:4** **(1)** Circle T for true or F for false.
**(a)** 9 thousandths + 5 hundredths
> 3 hundredths + 2 tenths          T   F
**(b)** 92 hundredths + 4 thousandths
> 0.924          T   F
**(c)** 0.456 < 0.5          T   F

**(2)** Write each number in the requested form.
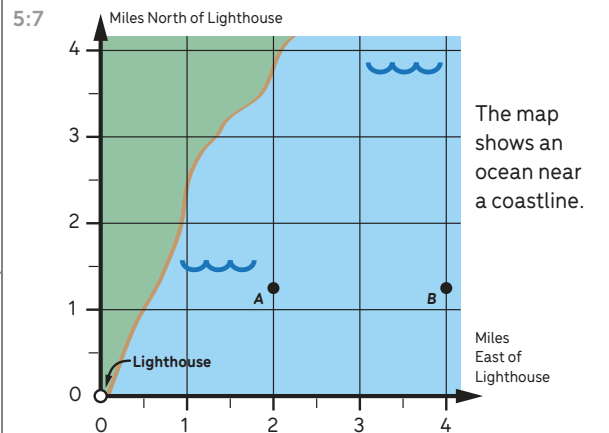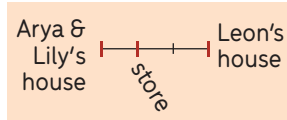**(a)** 7 thousandths + 5 tenths = _____ (decimal)
**(b)** 0.1 tenths = _____ (decimal)
**(c)** $\frac{2}{100} + \frac{5}{1000}$ = _____ (decimal)
= _____ (fraction in lowest terms)

**5:5** Write the requested values.

$4087 \times 53 = ?$      $\frac{1}{10} \div 10 = ?$      $0.4 \times 0.9 = ?$

$246 \times 914 = ?$      $\frac{7}{8} \times \frac{5}{3} = ?$      $0.75 \div 0.01 = ?$

$9744 \div 12 = ?$      $8 \times ? = 73$      $0.63 \div 0.3 = ?$

$1461 \div 6 = ?$      $3 \div \frac{1}{8} = ?$      $0.86 + 0.4 = ?$

$4 - (8 - 4) = ?$      $\frac{1}{2} + \frac{1}{3} - \frac{1}{5} = ?$      $0.72 - 0.17 = ?$

$0.02 + 0.2 = ?$

$\frac{1}{3} \div (6 \times 5) = ?$      $0.8 - 0.55 = ?$

$637 - 1.31 = ?$

**5:6** **(1)** Arya and Lily's house is $\frac{1}{5}$ mile from the store.

Arya & Lily's house | store | Leon's house

**(a)** Arya ran $\frac{1}{3}$ of the way from her house to the store. How far, in miles, did Arya run? **(b)** Lily ran $\frac{2}{3}$ of the way from her house to the store. How far, in miles, did Lily run? **(2)** It is $\frac{2}{5}$ mile from Leon's house to the store. **(a)** Leon ran $\frac{1}{3}$ of the way from his house to the store. How far, in miles, did Leon run? **(b)** Compare how far Leon and Lily ran; what do you notice, and why is it true?

**5:7**

Miles North of Lighthouse



Lighthouse

Miles East of Lighthouse

The map shows an ocean near a coastline.

Shipwrecks are at locations A $(2, 1\frac{1}{4})$ and B $(4, 1\frac{1}{4})$. Shipwrecks are also at locations C $(4, 3\frac{1}{2})$ and D $(2, 3\frac{1}{2})$. **(1)** Mark C and D on the map and shade rectangle ABCD. **(2)** Some believe there is sunken treasure in the region you shaded. How large is that region in mi²?

**5:8** A *scalene triangle* is a triangle in which the sides all have different lengths. Thinking about this, Alana decided there should also be a name for quadrilaterals in which the sides all have different lengths. She said, "I'll name them after myself." She defined an *alana-gon* to be a quadrilateral in which the four sides all have different lengths. **(1)** Draw an example of an alana-gon. **(2)** True or false: **(a)** All squares are alana-gons. **(b)** No trapezoids are alana-gons.

**5:9** On Saturday there was a walkathon.

I walked $\frac{1}{3}$ mile farther than Leslie.

Catherine

I walked $1\frac{1}{4}$ mile.

How many miles did Leslie walk?

**5:10** **(1)** Solve: $\frac{1}{3} = 0.1 + ?$
**(2)** Is there a number greater than $\frac{1}{5}$ and less than $\frac{1}{4}$? If you think so, find such a number. If you think there is no such number, explain why.
**(3)** Show one of the above problems and its solution on a number line.

**5:11** Juliet said, "I'm thinking of a rectangle. Its area is 1 square unit. Its perimeter is more than 1 million units."
**(1)** Is Juliet thinking of something possible or impossible? Use math to decide for sure.
**(2)** Explain your reasoning to your classmates. Revise your explanation based on suggestions from your classmates.

**5:12** Before it rained, the teacher went outside and placed identical baking pans on the ground. After it rained, the teacher brought the pans inside, and students measured how much water was collected in each pan.

Water Collected (liters)

0          1          2

If all the water collected were shared equally among the pans, how much water would be in each pan?

**5:13** In a snack shop there is a frozen yogurt machine. When there is 3 *l* of frozen yogurt in the machine, the machine is $\frac{1}{3}$ full. How much frozen yogurt is in the machine when it is $\frac{1}{4}$ full?

**5:14** Brandon was reading his math book. He saw the equation $\frac{3}{4} \times (4 + \frac{1}{2}) = 3 + \frac{3}{8}$. He said, "I don't get it—where did the 3 and the $\frac{3}{8}$ come from?" Write an explanation that could answer Brandon's question.

# Math Milestones — Grade 6

**6:1** $\frac{2}{3}$ of a charging cord is $\frac{1}{2}$ meter long. How long is the charging cord? (Answer in meters.)

**6:2** **(1)** Would you prefer 33% of a $100 prize or 75% of a $50 prize? **(2)** 8 is 25% of what number? **(3)** 14 is what percent of 200? **(4)** Write 6.25% as a decimal, then as a fraction in lowest terms. **(5)** Find the total cost of a $16 item after a sales tax of 6.25% is added. **(6)** A 3% tax on a $100 item adds ___ dollars to the cost. A 3% tax on a $1 item adds ___ dollars to the cost.

**6:3** The table shows temperatures at the South Pole before and after midnight on October 10–11, 2019.

Plot the data on graph paper and label the plot. Describe any patterns you see.
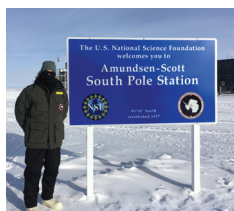
| Time | Hours after Midnight | Temp °F |
|---|---|---|
| 8:00 pm | −4 | −42 |
| 9:00 pm | −3 | −42 |
| 10:00 pm | −2 | −41 |
| 11:00 pm | −1 | −40 |
| Midnight | 0 | −39 |
| 1:00 am | 1 | −39 |
| 2:00 am | 2 | −38 |

**6:4** My car drives 570 mi with 15 gal of gas.
**(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I'll use ___ gal. **(b)** If I drive 5,700 mi, I'll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3) (a)** If I drive 532 mi, I'll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

**6:5** **(1)** Which of the numbers 5, −7, $\frac{2}{3}$, −$\frac{1}{2}$ is farthest from 0 on a number line? Which is closest to 0? **(2)** True or False: $\frac{1}{2} > −8$. **(3)** Explain why $−(−0.2) = 0.2$ makes sense.

**6:6** A farmer uses a tractor to plant corn quickly in the springtime. The farmer plants 216 acres every 12 hours. Create a formula for the number of acres the farmer plants in $n$ hours.

**6:7** **(1)** Look up the 50 top songs on a music streaming service. Type each song's duration into a spreadsheet. **(2)** Write a sentence about the data giving a measure of center and a measure of variability. **(3)** Make a histogram of the data.* **(4)** Write a sentence describing the overall pattern of the distribution and any striking deviations from the overall pattern. **(5)** Imagine that one year from now, you go back online and repeat (1)–(4). In what ways would you expect the data distribution to look similar? What differences would you expect to see?
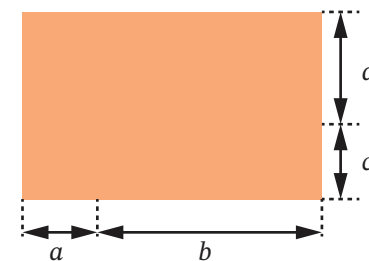
**Top 50 Songs Today**
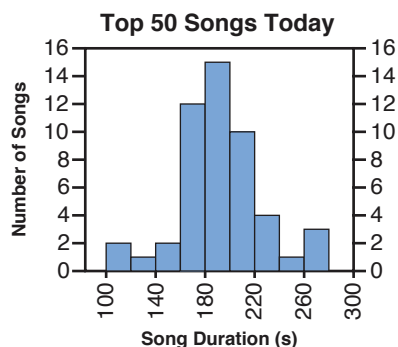
*Use this histogram for (4) and (5) if you don't do (3).

**6:8** *Pencils down* If $r = 1.748$, what is the value of $0.96r + 0.04r − r$?

**6:9** How much of a $\frac{3}{4}$-ton truckload is $\frac{2}{3}$ ton of gravel?

**6:10** In the month of February 2021, there were 20 weekdays and 8 weekend days. Here are some questions about that month. **(1)** (Circle all of the correct answers.) The ratio of weekdays to weekend days was 20:8  10:4  5:2  5:7. **(2)** There were ___ times as many weekdays as weekend days. **(3)** True or false: $\frac{5}{7}$ of the days that month were weekdays. **(4)** Approximately what percent of the days that month were weekdays?

**6:11** The diagram shows a rectangle. The variables $a$, $b$, $c$, and $d$ are lengths in meters. **(1)** Using the variables, write three different expressions for the area of the rectangle. **(2)** Choose two of your expressions and show that they are equivalent by applying properties of operations. **(3)** State the property or properties you used.

**6:12** **(1)** What is the area of the triangle in the coordinate plane with vertices $(1, 2)$, $(−5, 2)$, and $(−8, 9)$? **(2)** How does the area change if we change the third vertex to $(−3, 9)$?

**6:13** *Pencils down* Think about the equation $241p = \frac{3}{4}$. Is there a whole number that solves it? Is there a non-whole number that solves it? Convince a classmate that your answers are right.

**6:14** *Pencil and paper* **(1)** $81.53 \div 3.1 = ?$ **(2)** $\frac{7}{8} \div \frac{2}{3} = ?$ **(3)** Check both of your answers by multiplying.

**7:1** The cost of a phone is the phone's price, $264, plus 6.25% tax. **(1)** Use the expression $P + 0.0625 * P$ to find the cost. **(2)** Use the expression $P * 1.0625$ to find the cost. **(3)** Apply properties of operations to the expression $P + 0.0625 * P$ to produce the expression $P * 1.0625$.

**7:2** A utility pole 24 feet long has $28\frac{1}{4}$-inch circumference at the top and $47\frac{1}{8}$-inch circumference 6 feet from the base. Create and label a scale drawing of the pole in side view, with scale $\frac{1}{4}$ inch = 1 foot.

**7:3** Write each sum as a product with the given factor. *Example:* $8 + 6x = 2 \cdot ?$
*Answer:* $8 + 6x = 2(4 + 3x)$. **(1)** $6y + 12 = 3 \cdot ?$ **(2)** $-5w + 35 = (-5) \cdot ?$ **(3)** $4z + 1 = 4 \cdot ?$ **(4)** $9ay - 9by + 27cy = (9y) \cdot ?$

**7:4** *"Foul Play."* The Hawks were leading the Pistons in basketball by a score of 100-98. Just as time was running out, a Pistons player tried a 3-point shot. His defender had two choices: allow the shot, or stop it by fouling the Pistons player. Fouling would give the Pistons player 3 one-point free throws. The defender chose to foul and later wondered if it was a good choice. **(1)** To analyze the defender's choice, let's assume that for the Pistons player, every 3-point shot has probability $\frac{1}{3}$ of going in, and every free throw has probability 90% of going in. **(a)** If the defender allows the shot, what is the probability that the shot wins the game as time runs out? **(b)** If the defender stops the shot by fouling, estimate the probability that the free throws win the game. **(2)** Write a paragraph arguing for or against the defender's choice, based on probability calculations and/or simulations.

**7:5** *Pencils down* Think about the equation $x + 4\frac{1}{8} = \frac{2}{3}$. Is there a positive number that solves it? Is there a negative number that solves it? Tell how you decided.

**7:6**

**Car A** | **Car B**

distance (mi)



(1, 1.33)

distance (mi)

(1, 1)

time (min)
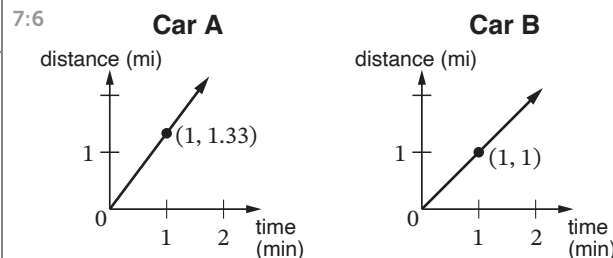
Car A and Car B were moving at constant speed, as shown in the graphs. **(1)** At the end of the first minute, how many miles had each car moved? **(2)** Which car was moving faster? **(3)** For the faster car, write a formula for the number of miles moved in $n$ minutes. **(4)** How many miles does the faster car move in 10 minutes?

**7:7** If the speed limit in Canada is 100 km/hr and you are driving 65 mph, are you over or under the limit? By how much?

**7:8** In 2018, an oil company rented an oil rig for $100,000 per day. The company drilled a well and started pumping oil. **(1)** How much oil must be sold each day to equal the rental cost? Note: 42 gal of oil could be sold for $70 in 2018. **(2)** The company estimates that the profit, $P$, in millions of dollars, after pumping oil for $D$ days is $P = 0.5D - 40$. **(a)** What is the profit after the first day of pumping oil? **(b)** Make a table of pairs of values $(D, P)$ and graph the ordered pairs. **(c)** How can the company make $30M of profit? **(3)** An equivalent expression for $P$ is $0.5(D - 80)$. How does the 80 in this expression relate to the company's situation?

**7:9** **(1)** Calculate. **(a)** $-4.1 + 4$ **(b)** $5 \div (-6)$ **(c)** $-1(-1 - 1)$ **(d)** $2 - (-\frac{1}{2})$ **(e)** $(-\frac{3}{8})(-8)$ **(f)** $0 - \frac{1}{3}$ **(g)** $\frac{1}{7.9} * 7.9$ **(h)** $(\frac{1}{2} - \frac{1}{4})(-9 + 9)$. **(2)** Show calculation 1(a) on a number line.

**7:10** In $\triangle ABC$, side AB is 4 units long, side BC is 3 units long, and angle A measures 30°. Sketch two ways $\triangle ABC$ might look.

**7:11** Nechama is shopping online for a ticket to a play. Website A offers a discount of $7.50 off the theater price. Website B offers a discount of 25% off the theater price. **(1)** Is it mathematically possible that Website A is a better deal than Website B? **(2)** Is it mathematically possible that Website B is a better deal than Website A? *Prove your answers.*

PALACE THEATER ADMIT ONE

**7:12** In 1972 in Loma, Montana, the temperature changed from −54°F to +49°F in a 24-hr period. Calculate the average rate at which the temperature changed. Answer to the nearest tenth in units of degrees/hr.

**7:13** A 15.1-in long wire is bent into the shape of a circle with 2.9 in left over. To the nearest 0.1 in, what is the diameter of the circle?

**7:14** Rose and Liba both solved this problem: *Jannat has 4 packs of balloons and 5 single balloons—29 balloons in all. How many balloons are in a pack?* Explain both of Rose's steps. Check that Liba's equations are all true statements about the balloons.

| **Rose** | **Liba** |
|---|---|
| $29 - 5 = 24$ | Let $x$ be the # of balloons in a pack. |
| $24 \div 4 = 6$ | $4x + 5 = 29$ |
| | $4x = 24$ |
| | $x = 6$ |

# Math Milestones — Grade 8

**8:1** Xavier's assignment for science class was to write notes to summarize a chapter in his textbook. At 4:45 p.m., he had 12 pages left to summarize. At 6:00 p.m., he had 7 pages left. Assuming a linear model, about how many more hours will it take him to finish summarizing?

**8:2** A pottery factory has two machines: a fast machine and a slow machine. The fast machine paints a pot in 3 min. The slow machine paints a pot in 10 min. Right now there's a pile of 50 unpainted pots waiting to go into the slow machine, and a pile of 28 unpainted pots waiting to go into the fast machine. **(1)** If you start the machines at the same time, which machine will finish its pile first? **(2)** How many min later will the other machine finish its pile? **(3)** Imagine instead that before starting the machines, you move some unpainted pots from the slow machine's pile to the fast machine's pile. How many pots would you move so that the two machines finish painting at the same time?

**8:3** On this blueprint for building a bike, part of the bike is shaped like a right triangle. The longest side length is illegible because water spilled on the blueprint. Calculate that side length.



30"  16"
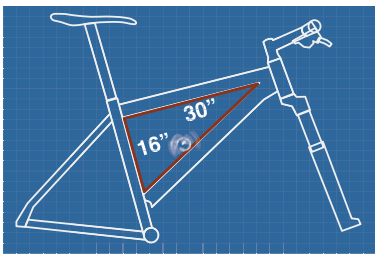
**8:4** **(1)** Decide whether each system has exactly one solution, infinitely many solutions, or no solutions. **(2)** For one system, justify your decision to your classmates in two ways: **(a)** drawing graphs of solutions; **(b)** algebraically.

$$\begin{cases} y = \frac{2}{3}x + 1 \\ y = \frac{2}{3}x + 2 \end{cases} \quad \begin{cases} d = 100 - 4t \\ d = 3.5 + t \end{cases} \quad \begin{cases} \frac{1}{8}Q + \frac{3}{8}R = -1 \\ Q + 3R = -8 \end{cases}$$

**8:5** Using physical models, transparencies, or geometry software, illustrate the fact that *rotations take angles to angles of the same measure.*

**8:6** Write as a fraction in lowest terms: **(1)** $1.041\overline{6}$. **(2)** $3^2 \cdot 3^{-5}$.

**8:7**

### City-to-City Distances & Airline Flight Times

| City-to-city distance (mi) | Flight time (hr) |
|---|---|
| 200 | 1.0 |
| 300 | 1.2 |
| 400 | 1.4 |
| 500 | 1.6 |

**(1)** How does flight time between cities depend on city-to-city distance? Answer by creating a function equation that models the data in the table. **(2)** Use your function to answer: **(a)** What is the time of flight if two cities are 1,000 mi apart? **(b)** What is the city-to-city distance if the flight took 2 hr? **(3)** Use your function and a spreadsheet to extend the table.

**8:8** A researcher asked people doing exercise to rate their effort level. The researcher also measured people's heart rates. Data were taken on two different days. **(1)** Use technology to plot the data from both days. (View heart rates in a window from 145 to 175.) Describe the main patterns you see. **(2)** On one of the days, the exercise room was warm, and on the other day, the room was cool. Which day do you think was the warm day? Tell how you decided, and support your answer with calculations.

### Heart Rate & Effort in Exercise

| Day 1 HR, Effort | Day 2 HR, Effort |
|---|---|
| 150.9, 1.3 | 148.6, 1.6 |
| 155.2, 1.5 | 152.7, 1.9 |
| 158.5, 1.8 | 153.9, 2.3 |
| 159.4, 2.1 | 155.4, 2.9 |
| 161.2, 2.1 | 156.6, 2.9 |
| 162.2, 2.3 | 157.9, 3.1 |
| 163.5, 2.4 | 158.9, 3.6 |
| 163.5, 2.7 | 159.7, 3.7 |
| 164.8, 2.7 | 160.6, 4.1 |
| 166.3, 2.9 | 161.3, 4.2 |
| 167.2, 3.0 | 162.3, 4.3 |
| 167.2, 3.3 | 162.4, 4.6 |
| 168.1, 3.4 | 163.4, 4.7 |
| 169.2, 3.4 | 164.2, 4.8 |
| 169.2, 3.5 | 164.8, 4.7 |
| 170.3, 3.5 | 165.0, 5.0 |
| 170.8, 3.6 | 165.4, 5.1 |
| 170.4, 3.7 | 167.0, 5.2 |
| 171.9, 3.7 | 166.5, 5.3 |
| 172.3, 3.9 | 166.7, 5.4 |

**Click here** to get the data online. Each person's heart rate (beats per min.) and effort (1–6 scale) were recorded every 3 min. A group average was then calculated, creating one data point such as (150.9, 1.3).

**8:9** A chef is cooking soup in a pot. If the chef keeps the soup gently boiling and doesn't cover the pot, water in the soup will evaporate. As water evaporates away, the soup will get thicker and tastier. Let's use a function equation to model the evaporation process: $D = 12 - 0.1t$. Variable $D$ is the depth of the soup in the pot, in units of cm, and variable $t$ is the amount of time the soup has been boiling, in units of min. **(1)** Graph the function. **(2a)** What is the value of the function for $t = 0$? **(2b)** What does your value in (2a) refer to in the situation? **(2c)** How is the situation at $t = 0$ represented on the graph? **(3)** What is the value of the slope of the graph, and what is the meaning of that value in the situation? **(4)** The soup is ready to eat when its depth is $\frac{2}{3}$ of the initial depth. At what time is the soup ready to eat? **(5)** Is the model useful for knowing what the depth of the soup would be at time $t = 150$ min? Why or why not?
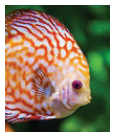
**8:10** Points A, B, and C lie on a straight line in the coordinate plane. By two methods, find the missing vertical coordinate.

C(6.0, 7.0)
B(5.2, **?** )
A(5.0, 5.0)
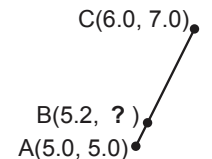
**8:11** Study a proof of the Angle-Angle criterion for triangle similarity. Explain one step of the proof in your own words.

**8:12** Design a fish tank that fits into the corner of a room. Use a quarter of a cylinder as a model for the tank. To share your design, make a diagram showing the tank measurements. Also, calculate the weight of the water when your tank is filled (1 m³ of water weighs about 1,000 kg). Write your calculation steps so that a classmate could understand how you did it.

# Doing Math at Home: K–2 Examples

## How might you use household items to practice math?

- Sort items by characteristics.
- Create repeating patterns.
- Count objects, focusing on one-to-one correspondence.
- Practice adding and subtracting objects.
- Create story problems with addition and subtraction.
- Measure lengths of objects.
- Write numbers to represent the quantity of objects.

\* Examples of household items: rocks, coins, milk lids, shells, beads, sticks, cotton balls, flowers, poker chips, small toys, macaroni, paper clips, pipe cleaners, yarn, pencils, straws, coffee stirrers, plastic toys, craft sticks, spaghetti noodles, shoes, cotton swabs, cookware, pretzel sticks, crayons, markers, dishes, bottle tops, etc.

## What could you do in the car to reinforce math skills?

- Count various objects (cards, trucks, motorcycles, mailboxes, stop signs, etc.).
- Rote counting.
- Practice math facts orally.
- Keep a white board and marker in the car to practice adding and subtracting.
- Identify shapes in road signs or architecture.
- Finger flashes: display fingers for 2–3 seconds and have the child say the number.
- Practice writing numbers and drawing pictures to represent the quantity.
- What Am I? Game:
  - Give clues about a number and have the child guess the number.
  - Give clues about the shape and have the child guess the shape.

## What can you do in the kitchen to incorporate math?

- Count food items or number of stirs.
- Practice comparison scenarios.
  - Which one has more? Less?
- Practice addition and subtraction with measurements or items.
- Practice partitioning food into equal shares, and discuss halves, thirds, and quarters.

# Doing Math at Home: 3–5 Examples

| | |
|---|---|
| | **How might you use household items to practice math?** |
| | <ul><li>Use items as data, create graphs, and create/answer questions about the data.</li><li>Estimate measurements of length, weight, and height.</li><li>Measure length, weight, and height in different units of measurement.</li><li>Create "groups of" to model multiplication.</li><li>Practice fair shares to model division.</li></ul> |

\* Examples of household items: rocks, coins, milk lids, shells, beads, sticks, cotton balls, flowers, poker chips, small toys, macaroni, paper clips, pipe cleaners, yarn, pencils, straws, coffee stirrers, plastic toys, craft sticks, spaghetti noodles, shoes, cotton swabs, cookware, pretzel sticks, crayons, markers, dishes, bottle tops, etc.

| | |
|---|---|
| | **What could you do in the car to reinforce math skills?** |
| | <ul><li>Keep a white board and marker in the car to practice multiplying and dividing.</li><li>Practice math facts orally.</li><li>Name two fractions and/or decimals, and have the child compare.</li><li>Identify shapes in road signs or architecture, and discuss attributes of the shape (for example: a square has four right angles, two pairs of parallel lines, and four equal sides).</li><li>Discuss distance to destination using fractions (1/2 way there, etc.).</li><li>What Am I? Game:<ul><li>Give clues about a number, and have the child guess the number.</li><li>Give clues about the shape, and have the child guess the shape.</li></ul></li></ul> |

| | |
|---|---|
| | **What can you do in the kitchen to incorporate math?** |
| | <ul><li>Practice measuring dry and liquid ingredients and ask questions about the fractions:<ul><li>How many halves are in one whole?</li><li>How can I measure 1 teaspoon if I only have a 1/4 teaspoon?</li><li>What's the difference between a pint and a quart?</li></ul></li><li>Practice partitioning food into equal shares, and discuss the fractions.</li><li>Calculate costs of items through repeated addition or multiplication.</li><li>Determine the area or perimeter of the kitchen.</li><li>Create a shopping budget, and involve the child in shopping to stay within budget.<ul><li>Practice adding and subtracting decimals with money.</li></ul></li></ul> |

# Doing Math at Home: 6–8 Examples

## How might you use household items to practice math?

- Use items as data, create graphs, and create/answer questions about the data.
- Measure objects, and practice converting between different units of measurement.
- Determine the volume and/or surface area of objects.
- Calculate the circumference of circular items.
- Use colored chips as positive and negative integers to practice adding/subtracting.
- Discussion ratios and proportions:
  - What's the ratio of the crayons to markers?

\* Examples of household items: rocks, coins, milk lids, shells, beads, sticks, cotton balls, flowers, poker chips, small toys, macaroni, paper clips, pipe cleaners, yarn, pencils, straws, coffee stirrers, plastic toys, craft sticks, spaghetti noodles, shoes, cotton swabs, cookware, pretzel sticks, crayons, markers, dishes, bottle tops, etc.

## What could you do in the car to reinforce math skills?

- Keep a white board and marker in the car to practice computation (multiplying and dividing fractions and decimals, percentages, etc.).
- Determine unit rates by calculating the price per gallon of gas.
- Discuss ratios by comparing cars to trucks, red cars to green, four wheels to two, etc.
- What Am I? Game:
  - Give clues about a number, and have the child guess the number. Be sure to include fractions, decimals, and percents.

## What can you do in the kitchen to incorporate math?

- Determine how many different ways to make a sandwich using identified ingredients (two meats, three types of cheese, three condiments, etc.).
- Convert between different units of measurement.
- Determine the volume/surface area of kitchen containers. Decide which one will hold the leftovers.
- Discussion ratios and proportions (red apples to green, sugar to flour, milk to water, etc.).
- Determine the cost of items with a certain percentage off.
- Calculate tax, tip, and new totals when eating out.
- Change a recipe to increase or decrease the number of servings.